

'Sic Bo' and 'Big and Small' Solutions

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Preliminaries

Recall that there are 216 possible outcomes. For most questions shown below, in part (a) the aim is to determine how many of these outcomes are favourable.

Sometimes it will be useful to refer to the ordered triple (a, b, c) , which denotes obtaining the a , b and c from die A, B and C respectively.

Question 1 – Specific Triple

By symmetry, the probability of winning is the same for any 'Specific Triple' bet. That is, the probability of getting 3 ones is the same as the probability of getting 3 sixes. Hence the calculations are identical for any 'Specific Triple' bet. This symmetry property applies for many of the available bets, so we won't mention this each time.

- (a) Only 1 of the possible 216 outcomes is the winning specific triple, so the probability of this bet winning is $\frac{1}{216}$.
- (b) The probability that a Specific Triple will win is $\frac{1}{216}$. If it wins the gambler receives back the \$1 stake and a further \$180 profit. The probability the bet loses is $\frac{215}{216}$, in which case the gambler loses the \$1 stake.

$$\text{Expected profit} = \frac{1}{216} \times \$180 + \frac{215}{216} \times (-\$1) = -\$ \frac{35}{216}$$

- (c) Expected profit = $\frac{1}{216} \times \$190 + \frac{215}{216} \times (-\$1) = -\$ \frac{25}{216}$

Question 2 – Specific Double

- (a) There are 3 ways to choose which die does not show the nominated number. This die may show any of the 5 other numbers. This gives 15 winning permutations. There is also the permutation where the chosen number appears on all 3 dice, giving 16 winning permutations.

For example, if the specific double selected was 6, the winning permutations would be:

$$\begin{aligned} &(6, 6, 1), (6, 6, 2), (6, 6, 3), (6, 6, 4), (6, 6, 5), (6, 6, 6) \\ &(6, 1, 6), (6, 2, 6), (6, 3, 6), (6, 4, 6), (6, 5, 6) \\ &(1, 6, 6), (2, 6, 6), (3, 6, 6), (4, 6, 6), (5, 6, 6) \end{aligned}$$

Thus the probability of this bet winning is $\frac{16}{216} = \frac{2}{27}$

(b) Expected profit = $\frac{2}{27} \times \$11 + \frac{25}{27} \times (-\$1) = -\$ \frac{1}{9}$

Question 3 – Any Triple

(a) There are 6 favourable outcomes, being the triples ranging from (1,1,1) to (6,6,6).

Probability of this bet winning = $\frac{6}{216} = \frac{1}{36}$

(b) Expected profit = $\frac{1}{36} \times \$31 + \frac{35}{36} \times (-\$1) = -\$ \frac{1}{9}$

(c) Expected profit = $\frac{1}{36} \times \$32 + \frac{35}{36} \times (-\$1) = -\$ \frac{1}{12}$

Question 4 – Two Dice Combination

(a) Symmetry applies. For ease of explanation, consider a bet on the domino combination “1 and 2”. For the winning cases involving 3 different numbers, the 3rd number can be chosen in 4 ways (3, 4, 5 or 6), and then the 3 numbers can be arranged in 3! ways. For the winning cases involving only two different numbers, there are two ways to choose which number will repeat (1 or 2) and then 3 ways to choose the location of the unique number (die A, die B or die C). Hence the number of favourable outcomes is $4 \times 3! + 2 \times 3 = 30$.

Probability of this bet winning = $\frac{30}{216} = \frac{5}{36}$

(b) Expected profit = $\frac{5}{36} \times \$6 + \frac{31}{36} \times (-\$1) = -\$ \frac{1}{36}$

(c) Expected profit = $\frac{5}{36} \times \$5.50 + \frac{31}{36} \times (-\$1) = -\$ \frac{11}{72}$

Question 5 – Single Die Bet

(a) By symmetry, the number of favourable outcomes is the same no matter what number is nominated by the gambler. For convenience of explanation, assume the gambler nominates the number 6.

Let X be a random variable denoting the number of 6’s obtained on the 3 dice. $X \sim \text{Bin}(3, \frac{1}{6})$.

Hence

$$P(X = 0) = {}^3C_0 \left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$P(X = 1) = {}^3C_1 \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^2 = \frac{75}{216}$$

$$P(X = 2) = {}^3C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) = \frac{15}{216}$$

$$P(X = 3) = {}^3C_3 \left(\frac{1}{6}\right)^3 = \frac{1}{216}$$

Alternatively, we can count the favourable outcomes from first principles.

No 6’s: There are 5 possible outcomes for each die, being the numbers 1 to 5, giving $5 \times 5 \times 5 = 125$ favourable outcomes.

Exactly one 6: There are 3 ways to chose which die gives the 6, and then there are 5 possible numbers for each of the other two dice, giving $3 \times 5 \times 5 = 75$ favourable outcomes.

Exactly two 6’s: There are 3 ways to chose the die which doesn’t gives the 6, and then 5 possible numbers for that die, giving $3 \times 5 = 15$ favourable outcomes.

Exactly three 6’s: Clearly (6,6,6) is the only favourable outcome.

We can check the sum of these 4 cases gives 216 outcomes.

(b) Expected profit = $\frac{1}{216} \times \$12 + \frac{15}{216} \times \$2 + \frac{75}{216} \times \$1 + \frac{125}{216} \times (-\$1) = -\$ \frac{1}{27}$

(c) Expected profit = $\frac{1}{216} \times \$10 + \frac{15}{216} \times \$2 + \frac{75}{216} \times \$1 + \frac{125}{216} \times (-\$1) = -\$ \frac{5}{108}$

Question 6 – Big and Small

- (a) When rolling three dice, the possible totals range from 3 to 18. Split the outcomes into 2 sets, being those with totals of 10 or less and those with totals of 11 or more. Each outcome in the first set is of the form (a, b, c) with a total $a + b + c \leq 10$ and with $a, b, c \in \{1, 2, 3, 4, 5, 6\}$. Each such outcome can be matched up with an outcome from the second set of the form $(7 - a, 7 - b, 7 - c)$ with a total $21 - (a + b + c) \geq 11$. Hence the number of outcomes in the two sets are equal.

That is, by symmetry, 108 of the 216 possible outcomes have totals of 10 or less and 108 have totals of 11 or more.

- (b) Of the 108 cases with totals in the range 3 to 10, we exclude the results $(1, 1, 1), (2, 2, 2)$ and $(3, 3, 3)$, giving 105 winning outcomes. Thus the probability that a “Small” bet wins is $\frac{105}{216} = \frac{35}{72}$.

Similarly, of the 108 cases with totals in the range 11 to 18, we exclude the results $(4, 4, 4), (5, 5, 5)$ and $(6, 6, 6)$, so the probability that ‘Big’ wins is the same.

(c) Expected profit = $\frac{35}{72} \times \$1 + \frac{37}{72} \times (-\$1) = -\$ \frac{2}{72} = -\$ \frac{1}{36}$

Question 7 – Three Dice Total

- (a) The only way to get a total of 3 is $(1, 1, 1)$ and the only way to get a total of 18 is $(6, 6, 6)$. These two bets are already available under ‘Specific Triple’ so there’s no need to repeat them as ‘Three Dice Total’ bets.
- (b) Symmetry again. Each permutation of the form (a, b, c) can be matched up with a permutation of the form $(7 - a, 7 - b, 7 - c)$. Hence the number of outcomes summing to $a + b + c$ must equal the number summing to $21 - (a + b + c)$. Stated more simply, the number of outcomes summing to x must equal the number summing to $21 - x$.
- (c) Of the 216 possible outcomes, the number totalling t , where $t \in \{3, 4, 5, \dots, 18\}$ is the number of solutions to

$$\begin{aligned} x_A + x_B + x_C &= t \\ x_A, x_B, x_C &\in \{1, 2, 3, \dots, 6\} \end{aligned}$$

where for example x_A denotes the outcome from die A. This is an “integer partition” problem. Using a generating function, we can say

Number of solutions

$$= \text{Coefficient of } x^t \text{ in } (x^1 + x^2 + x^3 + \dots + x^6)^3$$

$$= \text{Coefficient of } x^{t-3} \text{ in } (x^0 + x^1 + x^2 + \dots + x^5)^3$$

$$= \text{Coefficient of } x^{t-3} \text{ in } \left(\frac{1-x^6}{1-x} \right)^3$$

$$= \text{Coefficient of } x^{t-3} \text{ in } (1-x^6)^3 (1-x)^{-3}$$

$$= \text{Coefficient of } x^{t-3} \text{ in } (1-3x^6+3x^{12}-x^{18})(1+{}^3C_1x+{}^4C_2x^2+\dots)$$

We can evaluate this for $t \in \{3,4,5,\dots,10\}$, with the totals for 11 to 18 being determined by the symmetry property from part (b). Here are the results.

t	Outcomes	Probability
3	1	$\frac{1}{216}$
4	3C_1	$\frac{3}{216} = \frac{1}{72}$
5	4C_2	$\frac{6}{216} = \frac{1}{36}$
6	5C_2	$\frac{10}{216} = \frac{5}{108}$
7	6C_4	$\frac{15}{216} = \frac{5}{72}$
8	7C_5	$\frac{21}{216} = \frac{7}{72}$
9	${}^8C_6 - 3$	$\frac{25}{216}$
10	${}^9C_7 - 3 \times {}^3C_1$	$\frac{27}{216} = \frac{1}{8}$

- (d) First consider a total of 8 as an example. Every outcome with a total of 8 is a partition of 8 identical objects into 3 ordered cells with all cells occupied. We need at least one object per cell since a die can't roll a zero. For example, we allow (6,1,1) but not (6,0,2).

To ensure all cells are occupied, first place 1 object in each cell. The remaining 5 objects can then be partitioned between the 3 cells without restriction. The number of partitions possible is $\frac{7!}{5!2!} = {}^7C_2$, which matches the table given above in part (c).

The above approach is easily applied to the totals of 4 to 8, giving the results running from 3C_1 to 7C_5 , as given in the table in part (c). The total of 3 is most easily done as a special case, since there's clearly only one solution.

If we carry out the above process for a total of 9, we need to partition 9 objects into 3 cells, all cells occupied, and we find this can be done in 8C_6 ways. The difficulty is that this process has included the partitions (7,1,1), (1,7,1) and (1,1,7), which can't occur since the die can't land 7. Excluding these 3 cases gives ${}^8C_6 - 3$ outcomes, again matching part (c).

If we carry out the partition for a total of 10, we get 9C_7 partitions, but have included even more impossible die results, such as (7,2,1) and (8,1,1). To count the number of incorrectly included cases, note that at most 1 cell could get more than 6 of the objects. There are 3 ways to choose that cell. Give that chosen cell 7 objects, and the other two cells one object each. The 10th object can then be placed in any of the 3 cells. Hence the correct number of partitions is ${}^9C_7 - 3 \times 3$.

- (e) A page of output from a spreadsheet is appears as Appendix A at the end of this document. The count of favourable outcomes for each possible total are consistent with the results given in (c) above and confirm the symmetry property discussed in part (b).

The spreadsheet is straightforward, except for the process of generating the outcomes. Formulae in Excel syntax are given below for row 2 of the spreadsheet, the first row of data given that row 1 is used for headings. The outcome number appears in column A, and the results for dice A, B and C appear in columns B, C, D, (which is unfortunate labelling). MOD and CEILING are Excel functions. Other spreadsheets may use different names or different syntax.

Here is my preferred method.

$$\text{Die A: B2} = \text{CEILING}(\$A2/36,1)$$

$$\text{Die B: C2} = 1+\text{MOD}(\text{CEILING}(\$A2/6,1)-1,6)$$

$$\text{Die C: D2} = 1+\text{MOD}(\$A2-1,6)$$

However, other methods are possible. Here are two approaches that involve some formulae using the results of other formulae. I find these harder to understand but some people seem to think this way naturally.

$$\text{Die A: B2} = \text{CEILING}(\$A2/36,1)$$

$$\text{Die B: C2} = \text{CEILING}((\$A2-36*(\$B2-1))/6,1)$$

$$\text{Die C: D2} = \text{CEILING}((\$A2-36*(\$B2-1)-6*(\$C2-1)),1)$$

$$\text{Die A: B2} = 1+(\$A2-\$D2-6*(\$C2-1))/36$$

$$\text{Die B: C2} = 1+\text{MOD}((\$A2-\$D2)/6,6)$$

$$\text{Die C: D2} = 1+\text{MOD}(\$A2-1,6)$$

Extension Question 1

For the first solution given above, show how you would adjust the formula to list the 6^4 possible outcomes arising from rolling four dice. (The solution appears below, after the solution to Question 11.)

(f)

Total	Expected Return	
	Sic Bo	Big and Small
4 or 17	$\frac{3}{216} \times \$62 + \frac{213}{216} \times (-\$1) = -\$ \frac{27}{216} = -\$ \frac{1}{8}$	$\frac{3}{216} \times \$64 + \frac{213}{216} \times (-\$1) = -\$ \frac{21}{216} = -\$ \frac{7}{72}$
5 or 16	$\frac{6}{216} \times \$31 + \frac{210}{216} \times (-\$1) = -\$ \frac{24}{216} = -\$ \frac{1}{9}$	$\frac{6}{216} \times \$32 + \frac{210}{216} \times (-\$1) = -\$ \frac{18}{216} = -\$ \frac{1}{12}$
6 or 15	$\frac{10}{216} \times \$18 + \frac{206}{216} \times (-\$1) = -\$ \frac{26}{216} = -\$ \frac{13}{108}$	$\frac{10}{216} \times \$19 + \frac{206}{216} \times (-\$1) = -\$ \frac{16}{216} = -\$ \frac{2}{27}$
7 or 14	$\frac{15}{216} \times \$12 + \frac{201}{216} \times (-\$1) = -\$ \frac{21}{216} = -\$ \frac{7}{72}$	
8 or 13	$\frac{21}{216} \times \$8 + \frac{195}{216} \times (-\$1) = -\$ \frac{27}{216} = -\$ \frac{1}{8}$	
9 or 12	$\frac{25}{216} \times \$7 + \frac{191}{216} \times (-\$1) = -\$ \frac{16}{216} = -\$ \frac{2}{27}$	
10 or 11	$\frac{27}{216} \times \$6 + \frac{189}{216} \times (-\$1) = -\$ \frac{27}{216} = -\$ \frac{1}{8}$	

Question 8 – Even and Odd

- (a) Using the data from the previous question for totals of 2, 4, 6, ..., 18, the number of outcomes which give an even total is $3+10+21+27+25+15+6+1=108$, which is exactly half the outcomes. This is symmetry again. Each even total can be matched with an odd total that has exactly the same number of outcomes (3 with 18, 4 with 17, ...), so half of the 216 outcomes have an even total.

For an Even bet, we then exclude the 3 triples (2,2,2), (4,4,4) and (6,6,6), given 105 winning outcomes, and for the Odd bet we exclude the other 3 triples. Hence for both these bets, the probability of winning is $\frac{105}{216} = \frac{35}{72}$.

(b) Expected profit = $\frac{35}{72} \times \$1 + \frac{37}{72} \times (-\$1) = -\$ \frac{2}{72} = -\$ \frac{1}{36}$

That is, the probabilities and payout odds are the same as for the Big bet and the Small bet, so the expected profit is the same too.

Question 9 – Field Bet

(a) Number of favourable outcomes = $2 \times (6 + 10 + 15 + 21) = 104$

Required probability = $\frac{104}{216} = \frac{13}{27}$

(b) Expected profit = $\frac{13}{27} \times \$1 + \frac{14}{27} \times (-\$1) = -\$ \frac{1}{27}$

Question 10 – Four Number Combination Bet

(a) A winning outcome must contain 3 of the four numbers chosen. There are 4C_3 ways to choose which three numbers they will be, and then 3! ways to arrange the 3 numbers. This gives ${}^4C_3 \times 3! = 24$ favourable outcomes. Thus the probability that this bet wins is $\frac{24}{216} = \frac{1}{9}$.

(b) Expected profit = $\frac{1}{9} \times \$7 + \frac{8}{9} \times (-\$1) = -\$ \frac{1}{9}$

(c) ${}^6C_4 = 15$

Question 11

The results are summarised on the next page.

The least unfavourable bets, with an expected loss of 2.78%, are: Big, Small, Two Dice Combination (excluding SA), Even (SA only), Odd (SA only)

In South Australia, the most unfavourable bet, with an expected loss of 15.28%, is Two Dice Combination. In the other locations, the most unfavourable bet, with an expected loss of 16.20%, is Specific Triple.

Solution to Extension Question 1

Die A: B2 = CEILING(\$A2/216,1)

Die B: C2 = 1+MOD(CEILING(\$A2/36,1)-1,6)

Die C: D2 = 1+MOD(CEILING(\$A2/6,1)-1,6)

Die D: E2 = 1+MOD(\$A2-1,6)

Summary

Bet	Probability of winning	Payout for a win		Expected loss	
		Sic Bo	Big & Small	Sic Bo	Big and Small
Specific Triple	$\frac{1}{216} \approx 0.463\%$	180 to 1	190 to 1	$\frac{35}{216} \approx 16.20\%$	$\frac{25}{216} \approx 11.57\%$
Specific Double	$\frac{16}{216} = \frac{2}{27} \approx 7.407\%$	11 to 1		$\frac{1}{9} \approx 11.11\%$	
Any Triple	$\frac{6}{216} = \frac{1}{36} \approx 2.778\%$	31 to 1	32 to 1	$\frac{1}{9} \approx 11.11\%$	$\frac{1}{12} = 8.33\%$
Two Dice Combination	$\frac{30}{216} = \frac{5}{36} \approx 13.889\%$	6 to 1	5½ to 1	$\frac{1}{36} \approx 2.78\%$	$\frac{11}{72} \approx 15.28\%$
Single Die Bet	1 die: $\frac{75}{216} = \frac{25}{72} \approx 34.722\%$ 2 dice: $\frac{15}{216} = \frac{5}{72} \approx 6.944\%$ 3 dice: $\frac{1}{216} \approx 0.463\%$	12 to 1 2 to 1 1 to 1	10 to 1 2 to 1 1 to 1	$\frac{1}{27} \approx 3.70\%$	$\frac{5}{108} \approx 4.63\%$
Big or Small	$\frac{105}{216} = \frac{35}{72} \approx 48.611\%$	1 to 1		$\frac{1}{36} \approx 2.78\%$	
Three Dice Total:					
4 or 17	$\frac{3}{216} = \frac{1}{72} \approx 1.389\%$	62 to 1	64 to 1	$\frac{1}{8} = 12.50\%$	$\frac{7}{72} \approx 9.72\%$
5 or 16	$\frac{6}{216} = \frac{1}{36} \approx 2.778\%$	31 to 1	32 to 1	$\frac{1}{9} \approx 11.11\%$	$\frac{1}{12} = 8.33\%$
6 or 15	$\frac{10}{216} = \frac{5}{108} \approx 4.630\%$	18 to 1	19 to 1	$\frac{13}{108} = 12.04\%$	$\frac{2}{27} \approx 7.41\%$
7 or 14	$\frac{15}{216} = \frac{5}{72} \approx 6.944\%$	12 to 1		$\frac{7}{72} \approx 9.72\%$	
8 or 13	$\frac{21}{216} = \frac{7}{72} \approx 9.722\%$	8 to 1		$\frac{1}{8} = 12.50\%$	
9 or 12	$\frac{25}{216} \approx 11.574\%$	7 to 1		$\frac{2}{27} \approx 7.41\%$	
10 or 11	$\frac{27}{216} = \frac{1}{8} = 12.5\%$	6 to 1		$\frac{1}{8} = 12.50\%$	
Even or Odd	$\frac{105}{216} = \frac{35}{72} \approx 48.611\%$		1 to 1		$\frac{1}{36} \approx 2.78\%$
Field Bet	$\frac{104}{216} = \frac{13}{27} \approx 48.148\%$		1 to 1		$\frac{1}{27} \approx 3.70\%$
Four Number Combination Bet	$\frac{24}{216} = \frac{1}{9} \approx 11.111\%$		7 to 1		$\frac{1}{9} \approx 11.11\%$

Appendix A – Spreadsheet output for the Totals

Outcome number	Die A	Die B	Die C	Total	Total	Count
1	1	1	1	3	3	1
2	1	1	2	4	4	3
3	1	1	3	5	5	6
4	1	1	4	6	6	10
5	1	1	5	7	7	15
6	1	1	6	8	8	21
7	1	2	1	4	9	25
8	1	2	2	5	10	27
9	1	2	3	6	11	27
10	1	2	4	7	12	25
11	1	2	5	8	13	21
12	1	2	6	9	14	15
13	1	3	1	5	15	10
14	1	3	2	6	16	6
15	1	3	3	7	17	3
16	1	3	4	8	18	1
17	1	3	5	9	Check	
18	1	3	6	10	Sum	216
19	1	4	1	6		
20	1	4	2	7		
21	1	4	3	8		
22	1	4	4	9		
23	1	4	5	10		
24	1	4	6	11		
25	1	5	1	7		
26	1	5	2	8		
27	1	5	3	9		
28	1	5	4	10		
29	1	5	5	11		
30	1	5	6	12		
31	1	6	1	8		
32	1	6	2	9		
33	1	6	3	10		
34	1	6	4	11		
35	1	6	5	12		
36	1	6	6	13		
37	2	1	1	4		
38	2	1	2	5		
39	2	1	3	6		
40	2	1	4	7		
41	2	1	5	8		
42	2	1	6	9		
43	2	2	1	5		
44	2	2	2	6		
45	2	2	3	7		
46	2	2	4	8		
47	2	2	5	9		
48	2	2	6	10		
49	2	3	1	6		
50	2	3	2	7		
51	2	3	3	8		
52	2	3	4	9		
53	2	3	5	10		
54	2	3	6	11		

55	2	4	1	7
56	2	4	2	8
57	2	4	3	9
58	2	4	4	10
59	2	4	5	11
60	2	4	6	12
61	2	5	1	8
62	2	5	2	9
63	2	5	3	10
64	2	5	4	11
65	2	5	5	12
66	2	5	6	13
67	2	6	1	9
68	2	6	2	10
69	2	6	3	11
70	2	6	4	12
71	2	6	5	13
72	2	6	6	14
73	3	1	1	5
74	3	1	2	6
75	3	1	3	7
76	3	1	4	8
77	3	1	5	9
78	3	1	6	10
79	3	2	1	6
80	3	2	2	7
81	3	2	3	8
82	3	2	4	9
83	3	2	5	10
84	3	2	6	11
85	3	3	1	7
86	3	3	2	8
87	3	3	3	9
88	3	3	4	10
89	3	3	5	11
90	3	3	6	12
91	3	4	1	8
92	3	4	2	9
93	3	4	3	10
94	3	4	4	11
95	3	4	5	12
96	3	4	6	13
97	3	5	1	9
98	3	5	2	10
99	3	5	3	11
100	3	5	4	12
101	3	5	5	13
102	3	5	6	14
103	3	6	1	10
104	3	6	2	11
105	3	6	3	12
106	3	6	4	13
107	3	6	5	14
108	3	6	6	15
109	4	1	1	6
110	4	1	2	7
111	4	1	3	8
112	4	1	4	9

113	4	1	5	10
114	4	1	6	11
115	4	2	1	7
116	4	2	2	8
117	4	2	3	9
118	4	2	4	10
119	4	2	5	11
120	4	2	6	12
121	4	3	1	8
122	4	3	2	9
123	4	3	3	10
124	4	3	4	11
125	4	3	5	12
126	4	3	6	13
127	4	4	1	9
128	4	4	2	10
129	4	4	3	11
130	4	4	4	12
131	4	4	5	13
132	4	4	6	14
133	4	5	1	10
134	4	5	2	11
135	4	5	3	12
136	4	5	4	13
137	4	5	5	14
138	4	5	6	15
139	4	6	1	11
140	4	6	2	12
141	4	6	3	13
142	4	6	4	14
143	4	6	5	15
144	4	6	6	16
145	5	1	1	7
146	5	1	2	8
147	5	1	3	9
148	5	1	4	10
149	5	1	5	11
150	5	1	6	12
151	5	2	1	8
152	5	2	2	9
153	5	2	3	10
154	5	2	4	11
155	5	2	5	12
156	5	2	6	13
157	5	3	1	9
158	5	3	2	10
159	5	3	3	11
160	5	3	4	12
161	5	3	5	13
162	5	3	6	14
163	5	4	1	10
164	5	4	2	11
165	5	4	3	12
166	5	4	4	13
167	5	4	5	14
168	5	4	6	15
169	5	5	1	11
170	5	5	2	12

171	5	5	3	13
172	5	5	4	14
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182	6	1	2	9
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