

“Sic Bo” & “Big and Small” Questions

Version 0.1 Beta

8 September 2007

Copyright © 2007 Jim Farmer.

Permission is given to reproduce this document for educational purposes provided it is not altered in any way. “Educational purposes” includes both formal classes in probability and simply trying to convince a misguided individual that their “foolproof” system for beating the casino is not in fact foolproof.

This document is available from <http://gambling.plussed.net>

If you obtained it from some other source, you may not have the most up-to-date version.

Brief summary of rules

3 normal dice with faces numbered 1 to 6 are rolled inside a covered cage. Gamblers bet on the outcome, choosing from a restricted range of options. Then the cover is lifted to reveal the dice. Payout odds differ for the different available bets.

Regional Variations

Sic Bo is a legal method of throwing your money away in Casinos in the Australian Capital Territory and in the states of New South Wales, Queensland and Victoria. Some of the available bets have slightly different names between these locations. Also, in some cases, the wording in the relevant legislative instrument is vague or ambiguous. However, the intention appears to be that rules of the game are identical in all these locations.

South Australia has a very similar game called “Big and Small”. This game includes bets which are not available in Sic Bo as played elsewhere in Australia. It also has some bets which do appear in Sic Bo but which have a different payout odds in Big and Small, and there is no consistency as to whether the odds are better or worse than those for Sic Bo. The games are sufficiently similar that we will analyse them together. However, while the Approved Rules for South Australia refer to this game as Big and Small, presumably to highlight that it is different from Sic Bo as played outside South Australia, the web site of the Adelaide Casino seems to indicate that *they* call it Sic Bo. Odd indeed! Can this be legal?

Overseas readers should also note that payout odds frequently differ by country, and may differ by casino within a country. Also, you can't predict the odds in other countries by noting whether the game goes by the name ‘Sic Bo’ or ‘Big and Small’. The odds given below only apply to the Australian games of those names.

Traditionally the game involves the gamblers at a table making bets by placing chips on marked areas on the glass-topped table. After the dice are revealed, the human supervising the game enters the 3 numbers obtained into a device which causes all winning zones on the table top to light up. The supervisor then collects the chips on bets which lose and returns the chips and winnings to any gamblers who made winning bets.

NSW and Victoria also allow “Rapid Sic Bo”. This uses a computer system to record bets and pay winnings, if any. Each gambler has their own terminal in which they deposit cash and on which they enter their bets. The supervisor enters the winning numbers into a device and the computer system calculates any winning payouts and credits the accounts of the appropriate gamblers. This reduces the risk of human error in calculating payouts. More importantly, as the name implies, it greatly increases the number of games which can be played in a fixed time, ensuring that the

gamblers lose their money to the casino faster. It also reduces the Casino's wage bill, by allowing more gamblers to be involved in a single game. The traditional form only allows perhaps half a dozen gamblers per table, since the human supervisor must manually collect losing chips and pay winnings in chips. By contrast, Rapid Sic Bo eliminates the need for handling chips. The Victorian rules allow up to 50 gamblers at a single game, while the NSW rules allow up to 20.

Mathematical analysis

For the bets in Questions 1 to 7 the quotations of the rules are taken from the approved rules for NSW, which has the clearest explanation. The bets in Question 8 to 10 only occur in South Australia, so the quotations are from that state's approved rules.

We will call the dice, die A, die B and die C. Each die has 6 possible outcomes. Hence when the 3 dice are rolled there are $6 \times 6 \times 6 = 216$ equally likely possible outcomes. For each possible bet, we need to determine how many of these 216 outcomes result in a win.

(To produce equally likely outcomes, we assume the three dice are distinguishable. Whether or not the three dice are distinguishable to spectators is irrelevant.)

Question 1 – Specific Triple (ACT, NSW, Vic, SA) or Triple (Qld)

“Specific Triples” means a wager on any one of the specific triples 1, 2, 3, 4, 5, or 6, which shall win if that triple appears and shall lose if any other combination appears.’

- (a) Determine the probability that a particular specific triple bet wins.
- (b) In Sic Bo, the payout odds for a winning Specific Triple bet are 180 to 1. That is, if you bet \$1 and win, you receive your \$1 stake plus another \$180. Determine the expected profit for a \$1 bet. (The answer will be negative, since casino games of chance always favour the casino, not the gambler.)
- (c) In Big and Small the payout odds for winning this bet are 190 to 1. Determine the expected profit for a \$1 bet.

Question 2 – Specific Double (ACT, NSW, Vic) or Specific Double Dice Bet (SA) or Double (Qld)

“Specific Doubles” means a wager on any one of the specific doubles 1, 2, 3, 4, 5 or 6, which shall win if that double or win once only if a triple of the same number appears, and shall lose if any other combination appears.’

The ACT rules state: ‘Specific Double, which shall win if 2 of the 3 dice show the same nominated number’, which may leave one wondering whether the gambler still wins if all 3 dice show the chosen number. I assume the intention is that it does win, which is consistent with the rules for other locations in Australia.

- (a) Determine the probability that a particular Specific Double bet wins.
- (b) The payout odds for a winning Specific Double bet are 11 to 1. Determine the expected profit for a \$1 bet. (If we don't specifically refer to ‘Sic Bo’ or ‘Big and Small’, then the payout odds are identical in the two games.)

Question 3 – Any Triple

“Any Triple” means a wager on any triple 1, 2, 3, 4, 5 or 6 which shall win if a triple appears and shall lose if any other combination appears.’

- (a) Determine the probability that an Any Triple bet wins.

- (b) In 'Sic Bo' the payout odds for a winning an Any Triple bet are 31 to 1. Determine the expected profit for a \$1 bet.
- (c) In 'Big and Small' the payout odds for winning this bet are 32 to 1. Determine the expected profit for a \$1 bet.

Question 4 – Two Dice Combination (Vic), Dice Combinations (NSW, Qld), Domino Combination (ACT) or Domino Combination Bet (SA)

“Dice Combinations” of

1 and 2, 3, 4, 5 or 6,

2 and 3, 4, 5 or 6,

3 and 4, 5 or 6,

4 and 5 or 6,

5 and 6,

means a wager on any one of these specific combinations which shall win once only if that combination appears on two or more of the dice and shall lose if any other combination appears.’

More simply: you pick two different integers in the range 1 to 6 inclusive. If both numbers occur at least once die, you win.

The alternative name of Domino used in some locations arises because the symbols on the playing board for these bets resemble domino pieces. This isn't a good name, since the symbols for the Specific Double bets also resemble domino pieces. If we take all the Two Dice Combination symbols and the Specific Double symbols, we get all the domino pieces that use the numbers 1 to 6, and none of those using blanks.

The Queensland rules seem to contain a logic glitch. They state:

“dice combinations means a wager on 1 of the following combinations which wins if the combination appears and loses if any other combination appears”

and then goes on to list the same combinations as shown above from the NSW rules. Say a gambler bets on the combination “1 and 2”, and the three dice land 1, 2 and 3. The combination “1 and 2” has appeared, so s/he wins, but the other combinations “1 and 3” and “2 and 3” also appear, so s/he also loses. Oops! Presumably the intention is that the gambler would win, which would be consistent with the rules from the other locations.

- (a) Determine the probability that a particular Two Dice Combination bet wins.
- (b) In 'Sic Bo' the payout odds for winning this bet are 6 to 1. Determine the expected profit for a \$1 bet.
- (c) In 'Big and Small' the payout odds for winning this bet are 5½ to 1, a rare instance of non integer odds. (In the old days, we would have said 11 to 2.) Determine the expected profit for a \$1 bet.

Question 5 – Single Die Bet (NSW, Vic), Single Dice Bet (SA), Single Die (ACT), Individual Die Face Values (Qld)

“Single Die Bet” means a wager on any one of the numbers 1, 2, 3, 4, 5, or 6, which shall win if that number appears on one or more of the dice and shall lose if that number does not appear.’

However, if the bet wins, the payout odds differ depending on how many of the dice show the chosen number.

If the chosen shows on exactly one die, the payout is 1 to 1. If it shows on exactly two dice it is 2 to 1. If it shows on all three dice, the payout is 12 to 1 in Sic Bo and 10 to 1 in Big and Small.

- Determine the separate probabilities that the chosen number appears on exactly one, exactly two and all three dice. (It may also be useful to determine the probability that it doesn't appear, so that you can then check the 4 probabilities sum to 1.)
- Determine the expected profit for a \$1 bet in Sic Bo.
- Determine the expected profit for a \$1 bet in Big and Small.

Question 6 – Big and Small (ACT, NSW, Qld, SA, Vic)

“Small” means a wager which shall:

- win if any of the totals of 4, 5, 6, 7, 8, 9 or 10 appears in any combination of the three dice, with the exception of triple 2 or triple 3;
- lose if any other total or a triple appears.’

“Big” means a wager which shall:

- win if any of the totals of 11, 12, 13, 14, 15, 16 or 17 appears in any combination of the three dice, with the exception of triple 4 or triple 5;
- lose if any other total or a triple appears.’

In some locations, “Small” as described as winning if the total is in the range 3 to 10 inclusive, provided it is not a triple. Since a total of 3 can only occur as triple 1, this is equivalent to the above definition. Similarly “Big” may be described as covering totals of 11 to 18, excluding triples.

In the above quote, some section numbers have been replaced by bullet points to improve clarity.

- The possible totals from the 3 dice range from 3 to 18. Explain why the number of possible outcomes with totals of 10 or less is equal to the number with totals of 11 or more.
- Hence determine the probability of a ‘Big’ bet winning and the probability of a ‘Small’ bet winning.
- The payout odds for a winning a Big bet or a Small bet is 1 to 1. Determine the expected profit for a \$1 bet.

Question 7 – Three Dice Total (ACT, NSW, Vic), 3 dice total (Qld), Total Sum Combination (SA)

“Three Dice Totals” means a wager on any one of the totals 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 or 17 which shall win if that total appears in any combination of the three dice and shall lose if any other total appears.’

- A lateral thinking exercise: When rolling 3 dice, it is possible to get totals of 3 and 18. Why don't the Three Dice Total bets include these two totals?

And while these Three Dice Totals bets of 3 and 18 aren't allowed, we'll include them in the analysis below, since if we find the number of favourable outcomes for *every* possible total, we can check that all these favourable outcomes sum to the 216 possible outcomes.

- Explain why the probability of getting a total of x equals the probability of getting a total of $21 - x$.

This means we only have to count outcomes for the totals from 3 to 10, with 11 to 18 being deduced from symmetry. The next three parts give three different approaches for counting the outcomes for some or all of the totals.

- (c) For those who are familiar with generating functions: Use a generating function to find the number of favourable outcomes for each total from 3 to 10 and hence state the probability of that total. (If you prefer, use a probability generating function to get the probability directly.)
- (d) For those familiar with partitions of identical objects into ordered cells: Use this partition scenario to determine the favourable outcomes for the totals of 3 to 8. Explain why the method doesn't work nicely for totals of 9 and 10. For a real challenge, make it work for 9 and 10. (This approach is reasonable for Sic Bo but I don't recommend it for games involving more than 3 dice.)
- (e) Here is an alternative for those who don't know the techniques required in (c) and (d), but who are proficient with spreadsheets. This method is also probably the most useful if you are trying to explain the expected return to a non-mathematical person.

Use a spreadsheet to generate the full list of the 216 possible outcomes. Here are some hints.

Put the numbers 1 to 216 in the first column. Your aim is to list the 216 different possible outcome next to these numbers. For each such outcome place the result for dice A, B and C appear in the next three columns respectively. I suggest trying to list the outcomes in following the following order.

(1,1,1), (1,1,2), ..., (1,1,6), (1,2,1), (1,2,2), ... (1,6,6), (2,1,1), ..., (6,6,6)

It is possible to generate these outcomes with much copying and pasting. It is more efficient to use a formula for each of the columns B, C and D which generates the required die result from the outcome number in the first column. There are a few ways to do this. The most efficient ways make use of the modulus function (MOD in Excel) and/or functions to round up or down to an integer, (CEILING or FLOOR in Excel).

For each case, determine the total of the three dice, placing these results in column E.

Use an appropriate function (COUNTIF in Excel) to count the number of occurrences of each possible total in the data in column E.

- (f) For the Tree Dice Total bets, the payout odds differ by total, and some differ between Sic Bo and Big and Small. Given the data below, determine the expected profit for a \$1 bet in each game.

Total	Payout Odds	
	Sic Bo	Big and Small
4 or 17	62 to 1	64 to 1
5 or 16	31 to 1	32 to 1
6 or 15	18 to 1	19 to 1
7 or 14	12 to 1	
8 or 13	8 to 1	
9 or 12	7 to 1	
10 or 11	6 to 1	

The remaining bets are only available in South Australia. In the approved rules for South Australia, the descriptions of these bets include the label "Where available". The approved rules include two acceptable forms for the table layout, one of which has all of these extras and one of which has none of them. So, it seems a Casino in South Australia can choose to make all or none of these extra bets available.

Question 8 – Even and Odd (SA only)

'EVEN: (WHERE AVAILABLE) If the total of the 3 dice is even, the wager wins (with the exception of a triple).'

'ODD: (WHERE AVAILABLE) If the total of the 3 dice is odd, the wager wins (with the exception of a triple).'

- (a) There is a simple method to determine the probability that the total of the three dice will be even. If you can't see it, note that in the previous question we determined the probability of each total. Use those results to determine the probability of an even total and the probability of an odd total. Once you have done this, if necessary, kick yourself for not noticing the simpler method. Then exclude the appropriate triples to determine the probability that each of these bets wins.
- (b) The payout odds for a winning an Even bet or an Odd bet is 1 to 1. Determine the expected profit for a \$1 bet.

Question 9 – Field Bet (SA only)

'FIELD BET (WHERE AVAILABLE) The total of the numbers on the three dice drawn is either 5,6,7,8,13,14,15 or 16.'

(I have no idea why the approved rules print the parenthetic phrase in black when it's in red for the other extra bets.)

- (a) Use the results from Question 7 to determine the probability that this bet wins.
- (b) The payout odds for a winning a Field bet is 1 to 1. Determine the expected profit for a \$1 bet.

Question 10 – Four Numbers Combination Bet (SA only)

The version of the approved rules available online seems to have encountered a formatting issue which has jumbled the heading words with the explanation. Unjumbled, I think it intends to say:

'FOUR NO'S COMBINATION BET: (WHERE AVAILABLE) When the 3 numbers on the dice correspond to 3 out of 4 numbers on the betting position, the wager shall win.'

It isn't clear from this whether the four numbers may include duplicates. Consulting the table layout in the approved rules clarifies this. Only 4 different Four Number Combination Bets are available. They are: 6,5,4,3; 6,5,3,2; 5,4,3,2 and 4,3,2,1, an unpleasantly asymmetrical choice.

- (a) Determine the probability that a particular Four Numbers Combination Bet wins.
- (b) The payout odds for a winning Four Numbers Combination Bet are 7 to 1. Determine the expected profit for a \$1 bet.
- (c) As mentioned above, the playing table only allows four different Four Numbers Combination Bets, presumably because of space constraints. Assuming the four chosen numbers can't contain duplicates, how many different Four Numbers Combination Bets are theoretically possible if space wasn't an issue.

Question 11 – Best and worst bets

Clearly all bets are unfavourable to the gambler. Which bet or bets is least unfavourable? Which is most unfavourable?