Roulette Solutions

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Question 1 – Single Number

(a) Since there is only 1 winning number amongst the 37, the probability of winning is $\frac{1}{37}$.

(b) The probability that a particular single number bet will win is $\frac{1}{37}$. If it wins the gambler receives back the \$1 stake and a further \$35 profit. The probability the bet loses is $\frac{36}{37}$, in which case the gambler loses the \$1 stake.

Expected profit $= \frac{1}{37} \times \$35 + \frac{36}{37} \times (-\$1) = -\$\frac{1}{37}$

That is, the expected loss is $\frac{1}{37}$ of the outlay.

(c) Since there is only 1 winning number amongst the 36, the probability of winning increases to $\frac{1}{36}$ and the probability of losing reduces to $\frac{35}{36}$, giving

Expected profit $= \frac{1}{36} \times \$35 + \frac{35}{36} \times (-\$1) = 0$

Question 2 – Red or Black

(a) Whether we choose red or black, there are 18 winning number amongst the 37, so the probability of winning is $\frac{18}{37}$.

(b) The probability that a particular red or black bet will win is $\frac{18}{37}$. If it wins the gambler receives back the \$1 stake and a further \$1 profit. The probability the bet loses is $\frac{19}{37}$, in which case the gambler loses the \$1 stake.

Expected profit $= \frac{18}{37} \times \$1 + \frac{19}{37} \times (-\$1) = -\$\frac{1}{37}$

Curiously, the expected loss is that same as in Question 1(b).

(c) Since there are 18 winning numbers amongst the 36, the probability of winning increases to $\frac{18}{36} = \frac{1}{2}$ and the probability of losing reduces to $\frac{18}{36} = \frac{1}{2}$, giving

Expected profit $= \frac{1}{2} \times \$1 + \frac{1}{2} \times (-\$1) = 0$

Question 3 – General formulae for standard bets

(a) Since there are *n* winning number amongst the 37, the probability of winning is $\frac{n}{37}$.

(b) If the payout odds are *x* to 1, then on a 36 slot wheel there is a probability of $\frac{n}{36}$ of an $*x* profit and a probability of $1-\frac{n}{36}$ of a $1 loss, giving

$$\text{Expected profit in dollars} = \tfrac{n}{36}\times x + \left(1-\tfrac{n}{36}\right)\times(-1) = \frac{n(x+1)}{36} - 1$$

Equating the expected profit to zero gives

$$\frac{n(x+1)}{36} = 1$$
$$x = \tfrac{36}{n} - 1$$

This is consistent with the data in the table. For example, when $n=1$, implying a single number bet, the above equation gives $x = 35$, implying the payout odds are 35 to 1.

(c) The probability that the bet wins is $\frac{n}{37}$, in which case the profit in dollars is $x = \frac{36}{n}-1$. The probability that the bet loses is $1-\frac{n}{37}$, resulting in a $1 loss.

$$\text{Expected profit in dollars} = \tfrac{n}{37}\times\left(\tfrac{36}{n}-1\right) + \left(1-\tfrac{n}{37}\right)\times(-1) = -\tfrac{1}{37} \approx -2.703\%$$

The curious thing about this result is that it does not depend on *n*. The expected loss is 2.703% of the stake for all the standard bets.

(d) The probability of winning reduces to $\frac{n}{38}$.

$$\text{Expected profit in dollars} = \tfrac{n}{38}\times\left(\tfrac{36}{n}-1\right) + \left(1-\tfrac{n}{38}\right)\times(-1) = -\tfrac{1}{19} \approx -5.263\%$$

Again, the result is independent of *n*. The expected loss is 5.263% of the stake for all standard bets.

Question 4 - Neighbours

(a) Since there are 5 winning number amongst the 37, the probability of winning is $\frac{5}{37}$.

(b) If the bet loses, the $5 stake is lost. If it wins, $4 of the stake is lost, and the $1 which won is returned with a further $35 winnings, giving a profit of $31.

$$\text{Expected profit} = \tfrac{5}{37}\times\$31 + \tfrac{32}{37}\times(-\$5) = -\$\tfrac{5}{37} = -\tfrac{1}{37}\times\$5$$

That is, the expected loss is $\frac{1}{37}$ of the outlay, exactly the same as for any of the standard bets on a 37 slot wheel.

(c) Using the result of Question 1, the expected profit on each $1 straight up bet is $-\$\frac{1}{37}$, so the expected profit on a $5 neighbours bet is $5\times\left(-\$\frac{1}{37}\right) = -\$\frac{5}{37}$.

Question 5 – Combination Bets

(a) Let X_i be the random variable denoting the expected profit for the i^{th} component of the combination bet, which has outlay $\$a_i$. Question 3 proved $E\left(X_i\right) = -\frac{1}{37}a_i$. Hence the expected profit for the combination bet is

$$E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E\left(X_i\right) = \sum_{i=1}^{n} -\tfrac{1}{37}a_i = -\tfrac{1}{37}\sum_{i=1}^{n} a_i$$

That is, the expected loss for any combination bet is $\frac{1}{37}$ of the total outlay.

The remaining parts of this question are special cases of this general result, so they should also show the expected loss to be $\frac{1}{37}$ of the total outlay.

(b) The structure of this bet is identical to neighbours, the only difference being that is combines 3 straight-up bets rather than 5.

Expected profit $= \frac{3}{37}\times\left(\$35-\$2\right)+\frac{34}{37}\times\left(-\$3\right) = -\$\frac{3}{37} = -\frac{1}{37}\times\$3$

(c) The outlay is \$6. The 6 split bets cover 12 numbers, so there is a probability of $\frac{12}{37}$ that one of them will win. In this occurs, the split bet payout odds of 17 to 1 give winnings of \$17, but the other \$5 of the stake is lost.

The probability that none of the 12 numbers wins is $\frac{25}{37}$, in which case the \$6 stake is lost.

Expected profit $= \frac{12}{37}\times\left(\$17-\$5\right)+\frac{25}{37}\times\left(-\$6\right) = -\$\frac{6}{37} = -\frac{1}{37}\times\$6$

(d) The outlay is \$5.

The \$1 straight-up bet on 1 has a probability of $\frac{1}{37}$ of winning. If this occurs the payout odds of 35 to 1 give winnings of \$35, but the other \$4 of the stake is lost.

The 4 \$1 split bets covers 8 numbers, so there is a probability of $\frac{8}{37}$ that one of them will win. If this occurs, the winning split bet returns \$17, but the non-winning \$4 of the stake (which includes the \$1 straight up bet) is lost.

The probability that none of the 9 numbers wins is $\frac{28}{37}$, in which case the \$6 stake is lost.

Expected profit $= \frac{1}{37}\times\left(\$35-\$4\right)+\frac{8}{37}\times\left(\$17-\$4\right)+\frac{28}{37}\times\left(-\$5\right) = -\$\frac{5}{37} = -\frac{1}{37}\times\$5$

(e) The outlay is \$9.

The 5 \$1 split bets of 4/7, 12/15, 18/21, 19/22, 32/35 cover 10 numbers. The probability that one of these wins is $\frac{10}{37}$. If this occurs, the payout odds of 17 to 1 give winnings of \$17, but the other \$8 of the stake is lost.

The \$2 split bet on 25/29 covers 2 numbers. The probability that one of these wins is $\frac{2}{37}$. If this occurs, the payout odds of 17 to 1 give winnings of \$34, but the other \$7 of the stake is lost.

The \$2 street bet on 0/2/3 covers 3 numbers. The probability that one of these wins is $\frac{3}{37}$. If this occurs, the payout odds of 11 to 1 give winnings of \$22, but the other \$7 of the stake is lost.

The probability that none of the components of the combined bet win is $\frac{22}{37}$, in which case the \$9 stake is lost.

Expected profit

$$=\tfrac{10}{37}\times(\$17-\$8)+\tfrac{2}{37}\times(\$34-\$7)+\tfrac{3}{37}\times(\$22-\$7)+\tfrac{22}{37}\times(-\$9)=-\$\tfrac{9}{37}=-\tfrac{1}{37}\times\$9$$

(f) The outlay is \$4. The structure of this bet is very similar to "Orphans" from (d). Adjusting that part appropriately gives:

Expected profit $=\tfrac{1}{37}\times(\$35-\$3)+\tfrac{6}{37}\times(\$17-\$3)+\tfrac{30}{37}\times(-\$4)=-\$\tfrac{4}{37}=-\tfrac{1}{37}\times\$4$

(g) This case is sufficiently complex that we will revert to a table.

Winning Number	Calculations	Profit
13	1 \$4 corner bet returned, plus winnings at 8 to 1 of \$32. 1 \$6 six-line bet returned, plus winnings at 5 to 1 of \$30. The other \$30 of stake is lost.	\$32
14	1 \$2 split bet returned, plus winnings at 17 to 1 of \$34. 2 \$4 corner bets returned, plus winnings at 8 to 1 of \$64. 1 \$6 six-line bet returned, plus winnings at 5 to 1 of \$30. The other \$24 of stake is lost.	\$104
15	1 \$4 corner bet returned, plus winnings at 8 to 1 of \$32. 1 \$6 six-line bet returned, plus winnings at 5 to 1 of \$30. The other \$30 of stake is lost.	\$32
16	1 \$2 split bet returned, plus winnings at 17 to 1 of \$34. 1 \$3 street bet returned, plus winnings at 11 to 1 of \$33. 2 \$4 corner bets returned, plus winnings at 8 to 1 of \$64. 2 \$6 six-line bets returned, plus winnings at 5 to 1 of \$60. The other \$15 of stake is lost.	\$176
17	1 \$1 straight-up bet returned, plus winnings at 35 to 1 of \$35. 4 \$2 split bets returned, plus winnings at 17 to 1 of \$136. 1 \$3 street bet returned, plus winnings at 11 to 1 of \$33. 4 \$4 corner bets returned, plus winnings at 8 to 1 of \$128. 2 \$6 six-line bets returned, plus winnings at 5 to 1 of \$60. That is, the whole of the \$40 stake is returned.	\$392
18	1 \$2 split bet returned, plus winnings at 17 to 1 of \$34. 1 \$3 street bet returned, plus winnings at 11 to 1 of \$33. 2 \$4 corner bets returned, plus winning sat 8 to 1 of \$64.	\$176

	2 \$6 six-line bets returned, plus winnings at 5 to 1 of \$60. The other \$15 of stake is lost.	
19	1 \$4 corner bet returned, plus winnings at 8 to 1 of \$32. 1 \$6 six-line bet returned, plus winnings at 5 to 1 of \$30. The other \$30 of stake is lost.	\$32
20	1 \$2 split bet returned, plus winnings at 17 to 1 of \$34. 2 \$4 corner bets returned, plus winnings at 8 to 1 of \$64. 1 \$6 six-line bet returned, plus winnings at 5 to 1 of \$30. The other \$24 of stake is lost.	\$104
21	1 \$4 corner bet returned, plus winnings at 8 to 1 of \$32. 1 \$6 six-line bet returned, plus winnings at 5 to 1 of \$30. The other \$30 of stake is lost.	\$32

In all other cases, the \$40 stake is lost.

Expected profit $= \frac{4}{37}\times\$32+\frac{2}{37}\times\$104+\frac{2}{37}\times\$176+\frac{1}{37}\times\$392+\frac{28}{37}\times(-\$40)=-\$\frac{40}{37}=-\frac{1}{37}\times\$40$

Question 6 – A stray 38 slot wheel bet

(a) Since there are 5 winning numbers amongst the 38, the probability of winning is $\frac{5}{38}$.

(b) Expected profit $= \frac{5}{38}\times\$6+\frac{33}{38}\times(-\$1)=-\$\frac{3}{38}$

That is, the expected loss for a five-line bet is 7.895%, considerably worse than the 5.263% that applies to all other standard bets on a 38 slot wheel.

(c) On a 36 slot wheel we would find

Expected profit $= \frac{5}{36}\times\$6+\frac{31}{36}\times(-\$1)=-\$\frac{1}{36}\neq 0$

The difficulty is that 5, the number of winning numbers, does not divide evenly into 36, so the fair odds are not nice round numbers. The result of Q3(b) shows that the odds to break even on a 36 slot wheel would be $6\frac{1}{5}$ to 1, or 31 to 5. These odds are not conducive to easy calculation, so the nearest whole number odds of 6 to 1 were adopted.

Question 7 – Triplestar (NSW only)

(a) To win *only* twice in a row, the next throw must be 7 but the throw after that must be not 7, giving a probability of $\frac{1}{37}\times\frac{36}{37}$.

To win three times in a row, both of the next two throws must be 7, giving a probability of $\left(\frac{1}{37}\right)^2$. Note that we don't care whether the next throw after that is also 7.

(b) Expected profit $= \frac{36}{37}\times(-\$1)+\frac{1}{37}\times\frac{36}{37}\times(\$5-\$1)+\frac{1}{37}\times\frac{1}{37}\times\$1{,}005=-\$\frac{183}{1{,}369}$

An alternative expression which perhaps better reflects the order of play is

Expected profit $= \frac{36}{37}\times(-\$1)+\frac{1}{37}\times\left\{\$5+\frac{36}{37}\times(-\$1)+\frac{1}{37}\times\$1,000\right\}=-\$\frac{183}{1,369}$

That is the expected loss is approximately 13.167% of the outlay.

Question 8 – Double, Treble and Quad (Victoria only)

(a) Expected profit $= \frac{36}{37}\times(-\$1)+\frac{1}{37}\times\frac{36}{37}\times\$25+\frac{1}{37}\times\frac{1}{37}\times\$375=\frac{57}{1369}$

That is the expected loss is approximately 4.164% of the outlay.

(b) Working from first principles, the expected profit is

$$=\frac{36}{37}\times(-\$1)+\frac{1}{37}\times\frac{36}{37}\times\frac{1}{37}\times(-\$1)+\frac{1}{37}\times\frac{36}{37}\times\frac{36}{37}\times\$25$$
$$+\frac{1}{37}\times\frac{1}{37}\times\frac{36}{37}\times\$250+\frac{1}{37}\times\frac{1}{37}\times\frac{1}{37}\times\$5,000$$
$$=-\frac{2,920}{50,653}$$

Using the hint gives the much simpler solution:

Expected profit

$$=-\$1+\frac{1}{37}\times\frac{36}{37}\times\frac{36}{37}\times\$26+\frac{1}{37}\times\frac{1}{37}\times\frac{36}{37}\times\$251+\frac{1}{37}\times\frac{1}{37}\times\frac{1}{37}\times\$5,001$$
$$=-\frac{2,920}{50,653}$$

That is the expected loss is approximately 5.765% of the outlay.

(c) Expected profit $= \frac{36}{37}\times(-\$1)+\frac{1}{37}\times\frac{36}{37}\times\$25+\frac{1}{37}\times\frac{1}{37}\times\frac{36}{37}\times\$250+\frac{1}{37}\times\frac{1}{37}\times\frac{1}{37}\times\$5,000=-\frac{1,984}{50,653}$

That is the expected loss is approximately 3.917% of the outlay.

(d) This one is downright ugly from first principles, which gives the expected profit to be

$$=\left\{\frac{36}{37}+\frac{1}{37}\times\frac{36}{37}\times\frac{1}{37}+\frac{1}{37}\times\frac{36}{37}\times\frac{36}{37}\times\frac{1}{37}+\frac{1}{37}\times\frac{1}{37}\times\frac{36}{37}\times\frac{1}{37}\right\}(-\$1)$$
$$+\frac{1}{37}\times\left(\frac{36}{37}\right)^3\times\$12.5+\frac{1}{37}\times\frac{1}{37}\times\left(\frac{36}{37}\right)^2\times\$125$$
$$+\left(\frac{1}{37}\right)^3\times\frac{36}{37}\times\$12,500+\left(\frac{1}{37}\right)^4\times\$500,000$$
$$=-\$\frac{130,972}{1,874,161}$$

Using the hint gives the expected profit to be much nicer result

$$=-\$1+\frac{1}{37}\times\left(\frac{36}{37}\right)^3\times\$13.5+\left(\frac{1}{37}\right)^2\times\left(\frac{36}{37}\right)^2\times\$126+\left(\frac{1}{37}\right)^3\times\frac{36}{37}\times\$12,501+\left(\frac{1}{37}\right)^4\times\$500,001$$
$$=-\frac{130,972}{1,874,161}$$

That is the expected loss is approximately 6.988% of the outlay.

(e) Expected profit

$$=\frac{36}{37}\times(-\$1)+\frac{1}{37}\times\frac{36}{37}\times\$12.5+\left(\frac{1}{37}\right)^2\times\frac{36}{37}\times\$125$$
$$+\left(\frac{1}{37}\right)^3\times\frac{36}{37}\times\$12,500+\left(\frac{1}{37}\right)^4\times\$500,000$$
$$=-\$\frac{90,958}{1,874,161}$$

That is the expected loss is approximately 4.853% of the outlay.